# Adaptive Torque Controller Design for a Variable Stiffness Actuator.

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## Introduction

Variable stiffness actuator (VSA) has been widely implemented to rehabilitation devices. The mechanical stiffness is proportional to the closed-loop bandwidth, which leads to an advantage of VSA to achieve different tasks. For the controller design, a linear parameter-varying (LPV) system is formed due to the stiffness variation. Based on the LPV model, a gain-scheduled torque control approach is presented in this paper.

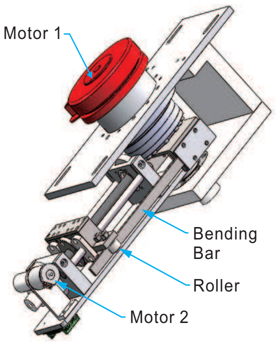


**Figure 1:** Mechanical design of the VSA

The actuator prototype is proposed in Fig. 1, where a double-motor driving system is achieved to control the joint (by Motor 1) and stiffness (by Motor 2) independently. The stiffness transmission is dependent on the controllable effective length of the bending bar by moving the rollers position, which can be classified as the adjustable lever-arm mechanism [1].

**Table 1:** Nomenclature and actuator specifications

| Symbol | Specification |
|---|---|
| $\theta_1$ | Motor 1 position |
| $\omega_1$ | Motor 1 speed |
| $\theta_2$ | Motor 2 position |
| $\theta_j$ | Joint position |
| $\gamma_1$ | Gear ratio |
| $T_j$ | Joint torque |
| $K_j(\theta_2)$ | Variable stiffness function |
| $T_{M_1}$ | Motor input torque |
| $J_1$ | Moment inertia of motor |
| $B_1$ | Friction constant of motor |
| **Actuator performance** | **Value** |
| Stiffness range | 200 - 500 Nm/rad |
| Weight | 3.1 kg |

To facilitate the derivation of the model, the parameters using in this paper are given in Table 1.

## Method and Material

### Preliminary Studies

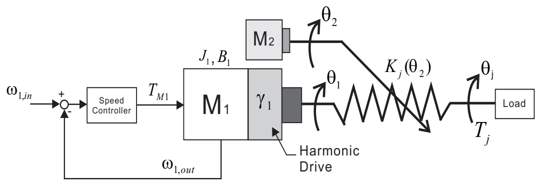


**Figure 2:** Control structure of the VSA: Open-loop torque control

Fig. 2 presents a cascaded control system including the VSA dynamics. The inner loop was set to be a speed control model with a Proportion-Integrate (PI) controller. For the controller design, the following assumptions were made:

1. For the simplification of LPV modelling, the variable parameter $K_j(\theta_2)$ is regarded as an exogenous signal, for which the coupled torque between the joint motion and stiffness variation are neglected.
2. For torque controller design, the fixed-load condition is generally provided (i.e. $\theta_j = 0$).

The dynamical equation of the VSA is then given by:

$$T_{M_1} = J_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + \frac{T_j}{\gamma_1} \tag{1}$$

where

$$T_j = K_j(\theta_2)\theta_1/\gamma_1 \tag{2}$$

was obtained by using Hooke's Law.

Combining Eq. (1) and (2), the cascaded system in Fig .2 can be written as a state-space LPV model with the variable parameter $K_j(\theta_2)$:

$$\begin{aligned} \dot{x} &= A(K_j(\theta_2))x + Bu \\ y &= C(K_j(\theta_2))x \end{aligned} \tag{3}$$

where (A, B, C) is the state-space realization, x = $[\omega_{1,out}, \theta_{1,out}, \theta_{1,in}]^T$, u = $T_{M_1}$ and y = $T_j$ are the state, input and output vector, respectively.

### Torque Controller Design

In our paper, the requirements for control performance are given by:

1. The closed-loop bandwidth should be larger than the natural frequency of human leg (i.e., 5 Hz).
2. Zero steady state error for torque tracking.
3. Robust stability with respect to the modeled dynamics.

A classical PI controller can be used to guarantee the stability for the torque control-loop. The goal in the controller design is to meet the requirement in the whole operating points. Therefore, a gain-scheduled PI controller based on the interpolation method (see [2]) is proposed.

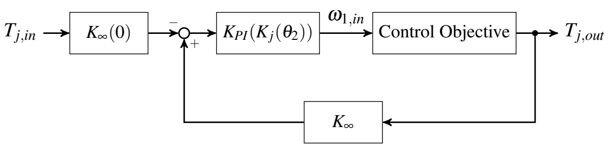


**Figure 3:** Closed-loop torque control

To guarantee robust stability for the closed-loop system, an $H_\infty$ loop-shaping controller is then synthesized [3]. The final control structure is given in Fig. 3, where the control objective is the open-loop system in Fig. 2, $K_\infty$ is the $H_\infty$ controller, $K_\infty(0)$ is a constant pre-filter and $K_{PI}(K_j(\theta_2))$ is the gain-scheduled PI controller.

### *Algorithm Example and Experiment*

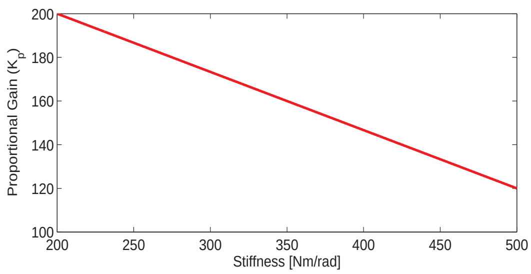


**Figure 4:** Stiffness versus proportional gain ($K_p$)

We first choose the minimum and maximal stiffness as the operating points to adjust the PI controller (denoted as a transfer function form, i.e., $K_P(K_j(\theta_2)) + K_I/s$). The integral gain, $K_I = 20$, was set to a constant due to the limited influence on bandwidth. A linear fitting function between stiffness and proportional gain is given in Fig. 4.

To satisfy the stability for the parameter-varying system, the problem comes to find a symmetric matrix, $X(K_j(\theta_2))$, such that [4]:

$$X(K_j(\theta_2))\hat{A}(K_j(\theta_2)) + \hat{A}(K_j(\theta_2))^T X(K_j(\theta_2)) + \dot{K}_j(\theta_2)\frac{\partial X(K_j(\theta_2))}{\partial K_j(\theta_2)} < 0 \tag{4}$$

where $\hat{A}(K_j(\theta_2))$ is the system A matrix of the closed-loop. Based on the actuator specifications, we have $K_j(\theta_2) \in [200,\ 500]$ Nm/rad and the maximum speed of stiffness variation is $\dot{K}_j(\theta_2) = 80$ Nm/rad/s. In such case, the closed-loop system can meet stability by solving the LMI in Eq. (4). For the robust controller design, a linear time invariant system is achieved by adjusting $K_j(\theta_2) = 350$ Nm/rad and $K_p = 160$ to be frozen. An reduced-order $H_\infty$ controller ($3^{th}$ order) is then obtained by solving two Riccati equations in Matlab.

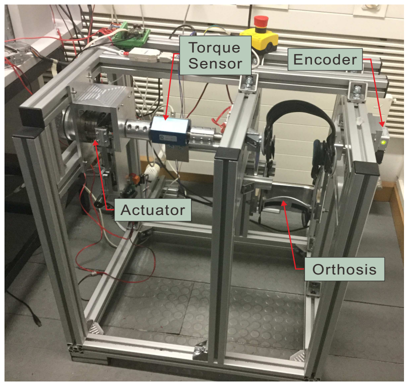


**Figure 5:** Prototype of the test bench

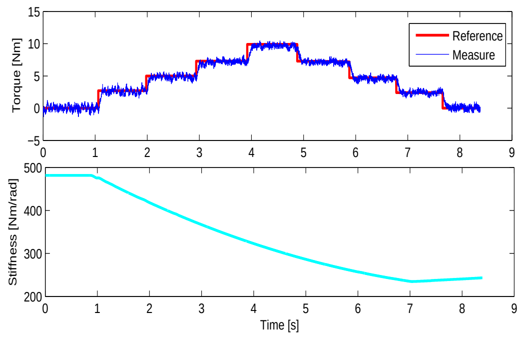


**Figure 6:** Square wave test of the closed-loop system

A hardware-in-the-loop test in dsPACE has been implemented to control the system on a test bench (see Fig. 5). A real-time square wave test with stiffness variation was used to excite to the system. Fig. 6 shows that the gain-scheduled system can achieve a good tracking performance.

## Conclusion

A gain-scheduled control approach has been achieved in this paper. For the future discussion, the bandwidth performance could be analyzed from different aspects, such as frequency response function and Human-in-the-loop test. Comparison with the LTI control system, the control method possess a better tracking performance and robustness.

## References


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